Area-Preserving Simplification of Polygon Features

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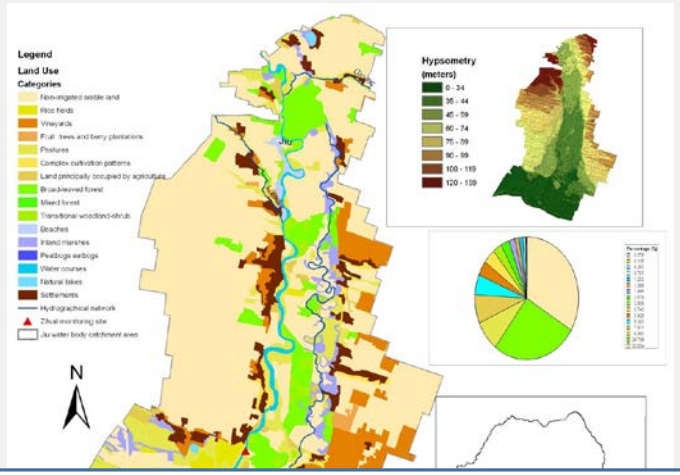
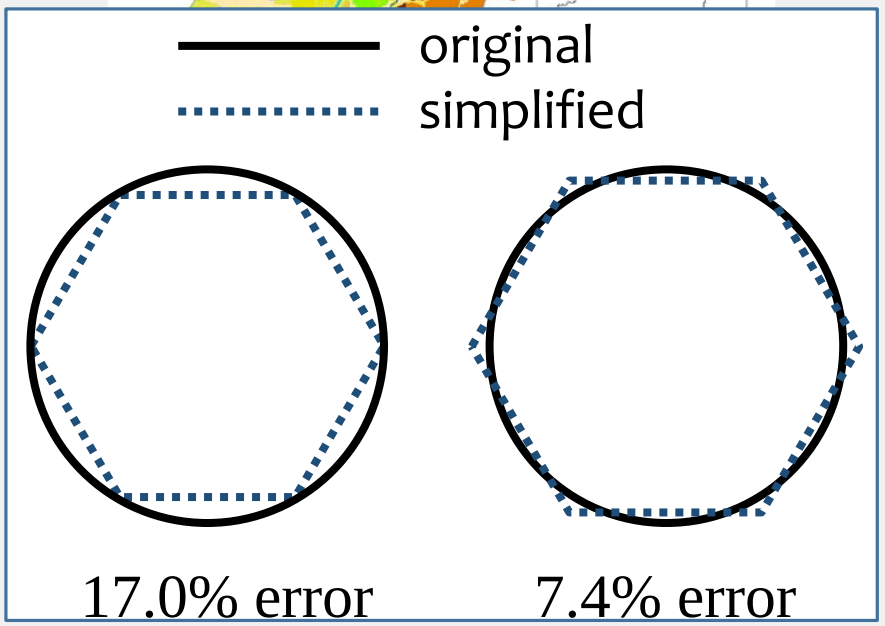
Motivation

- Area-based analysis
  - thematic density maps
  - hydrological models
  - cartograms
- Improved Representation
  - error balancing





Area-Preserving Feature Simplification

- requires Steiner points

  (Bose et al. 2006)

- previous algorithms
  - rectilinear schematization

    (Meulemans et al. 2010)

  - smoothing algorithm

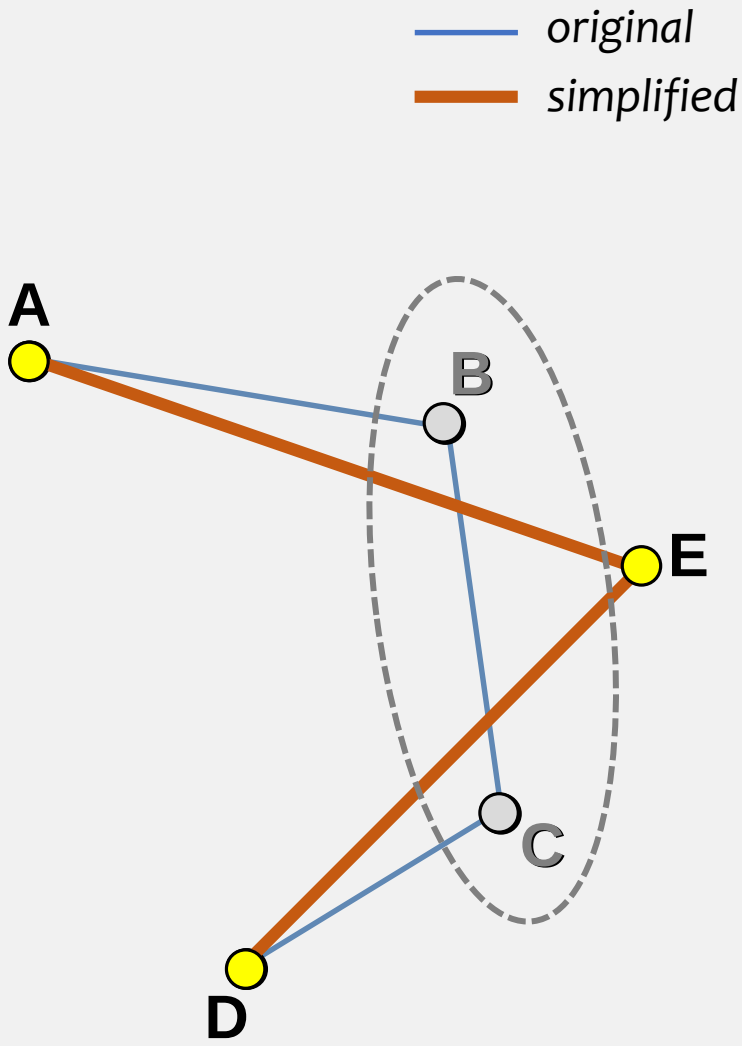
    (Tutić and Lapaine 2009)

Objectives

- Develop polygon simplification algorithm that:
  - preserves polygon area
  - minimizes displacement
  - is computationally efficient
- Model-based approach

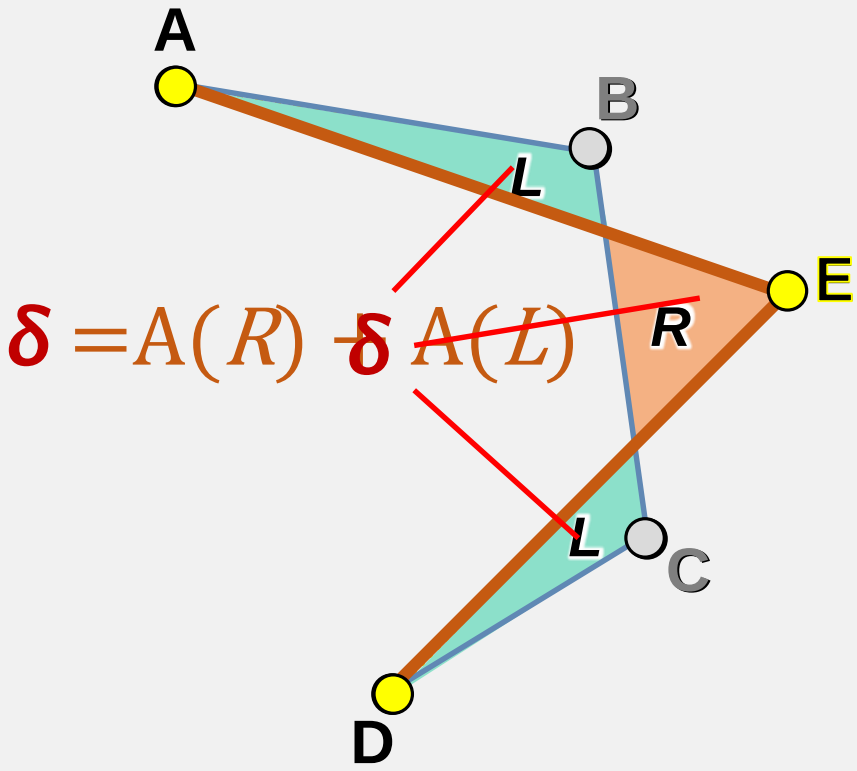
Segment Collapse

- begin with 4-vertex sequence {ABCD}
- collapse segment {BC} to point E



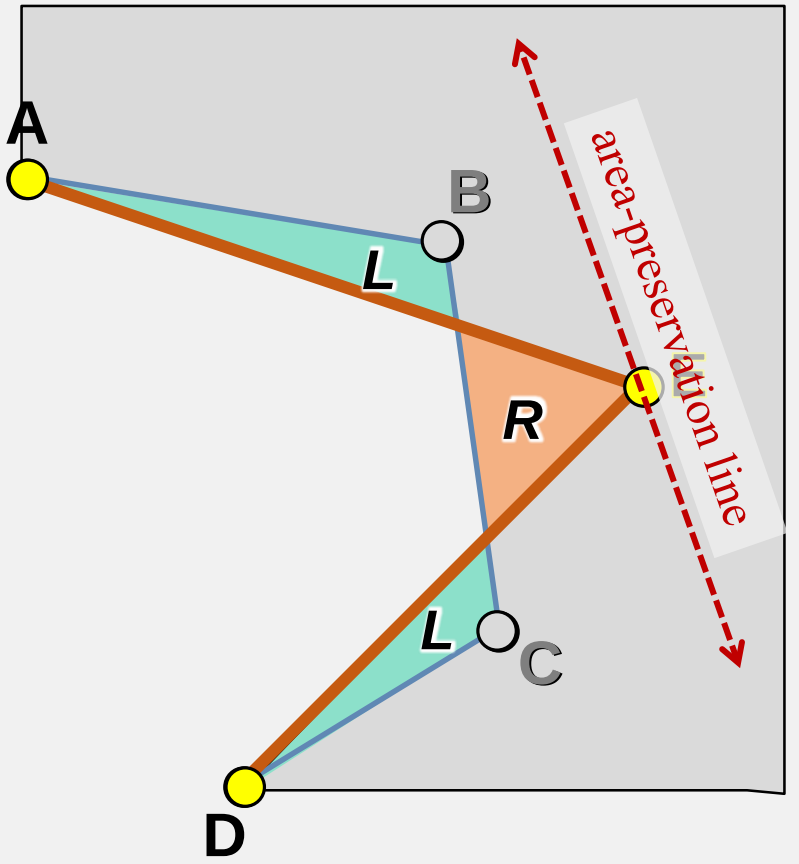
Displacement Function

- area between $\overline{ABCD}$ and $\overline{AED}$
- sum of right & left displacement areas:



Placement Function

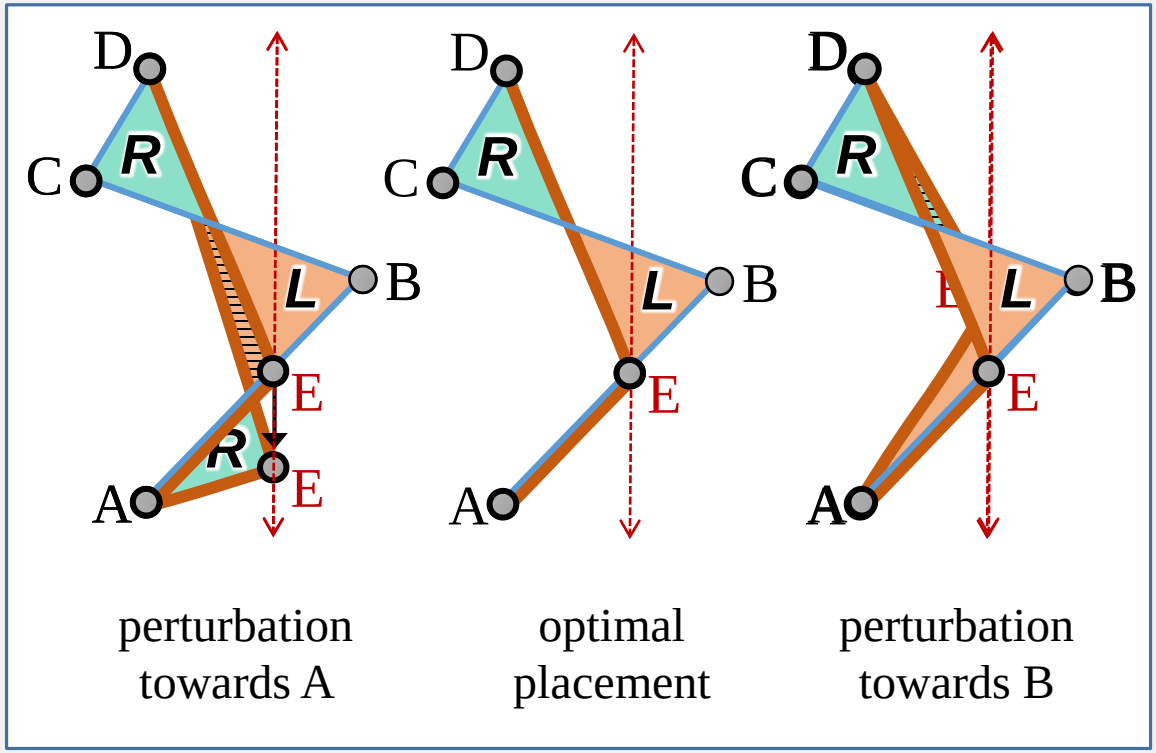
- Area Preservation Condition
  - $A(R) = A(L)$
- solution:
  - $ax_E + by_E + c = 0$
  - $a = y_D - y_A$
  - $b = x_A - x_D$
  - $c = -y_B x_A + (y_A - y_C)x_B + (y_B - y_D)x_C + y_C x_D$



Placement Function

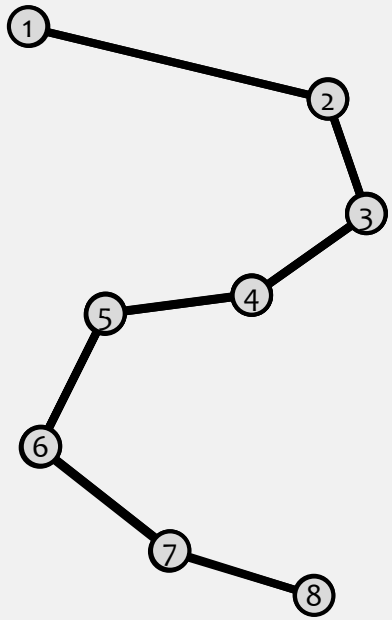
original
simplified

- Minimizing Areal Displacement
  - optimal placement is at intersection of $\overleftrightarrow{ap}$ and either $\overleftrightarrow{AB}$ or $\overleftrightarrow{CD}$
- Proof:
  - optimality can be proven logically from small perturbations

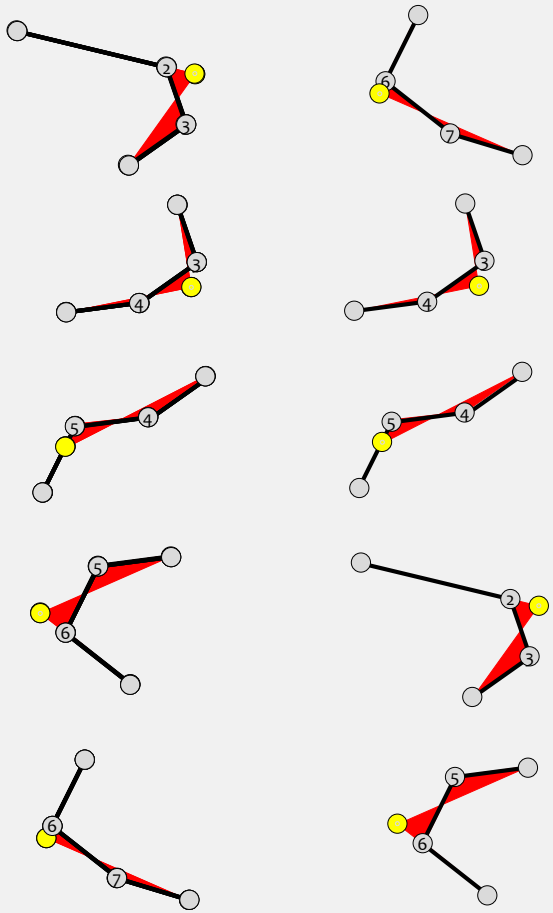


Segment Collapse Algorithm

1. Process all segments initially
2. *calculate placement (E)*
3. *calculate displacement (δ)*
4. *create worklist sorted by δ*



Work List



Segment Collapse Algorithm

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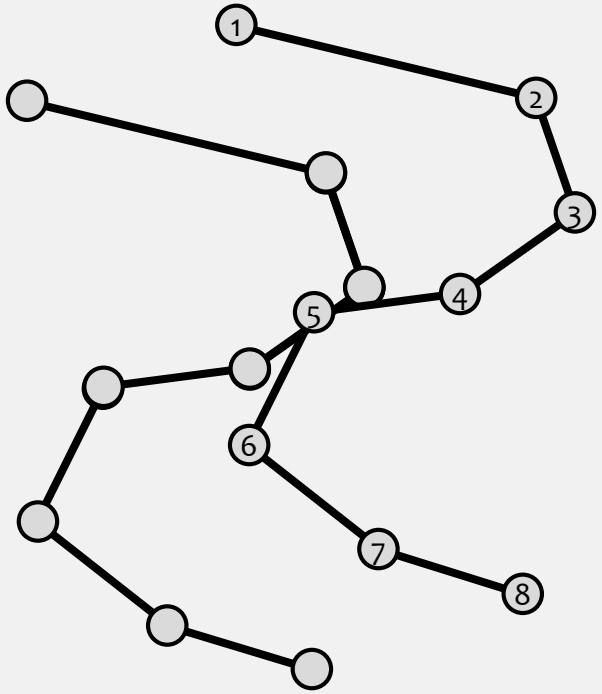
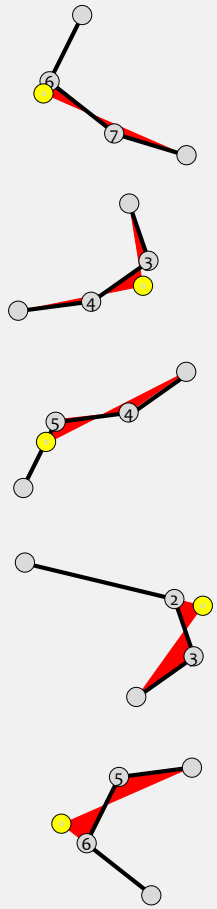
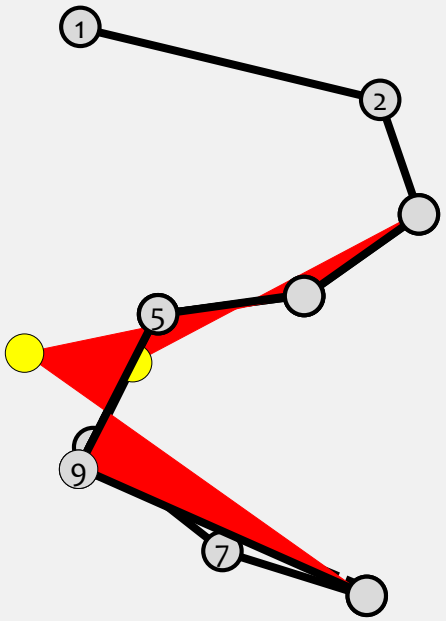


Work List

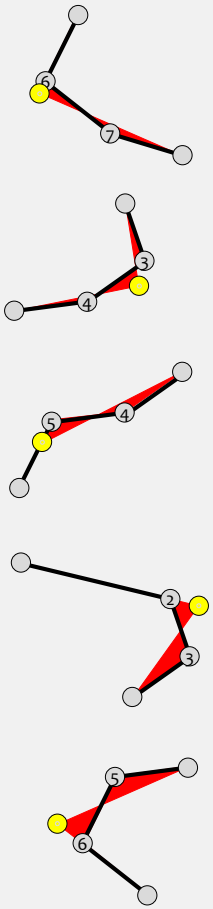


Segment Collapse Algorithm

1. Process all segments initially:
2. *calculate placement (E)*
3. *calculate displacement (δ)*
4. *create worklist sorted by δ*
5. Repeat:
6. *pull 1st segment from worklist*
7. *replace with new point (E)*
8. *remove obsolete segments*
9. *add newly created segments to worklist*

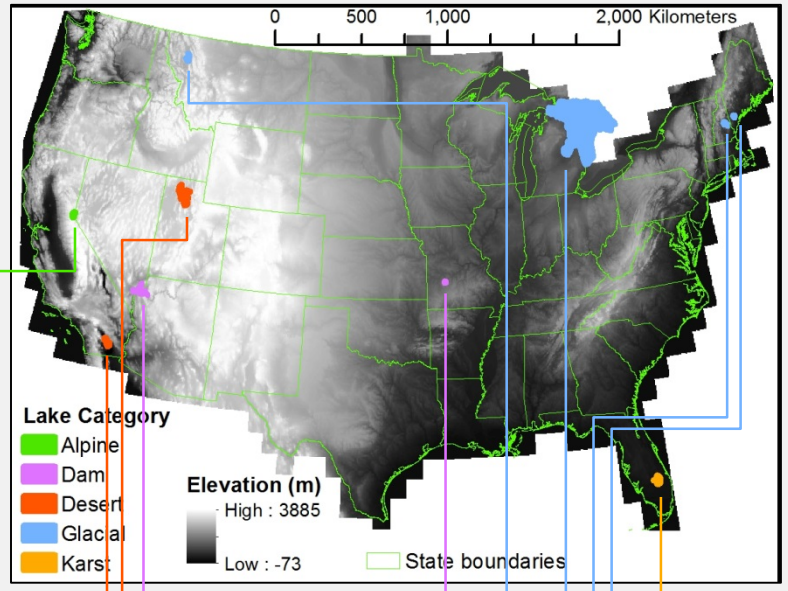


Work List



Algorithm Assessment

- Data:
  - high-res NHD
  - 10 sample lakes
  - 31 km² to 63,000 km²
  - five geomorphic settings



alpine *desert* *dam* *glacial* *karst*

Algorithm Assessment

Algorithms:

- *Visvalingam-Whyatt Effective Area*
- *Raposo Spatial Means*
- *Ramer-Douglas-Peucker*

Scales:

- *1:100,000*
- *1:500,000*
- *1:1,000,000*
- *1:5,000,000*

Vertices Retained*:

- *49%*
- *22%*
- *15%*
- *7%*

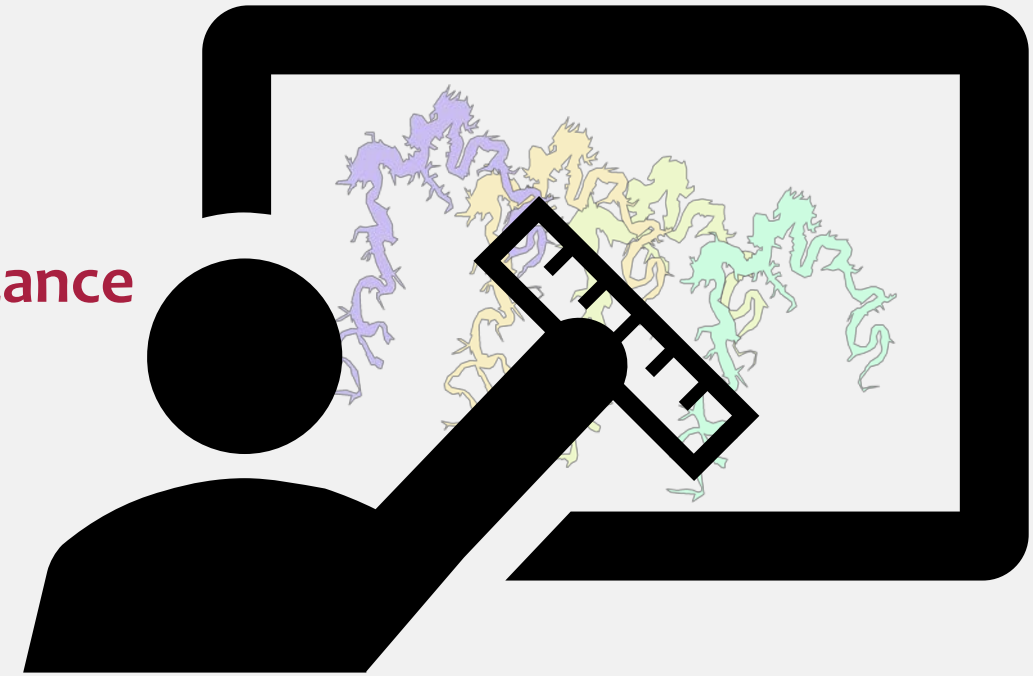
*Determined by Radical Law: Principle of Selection

(Töpfer and Pillewizer 1966)

$$\#\_items_{target} = .\#\_items_{source} * \sqrt{RF_{source} / RF_{target}}$$

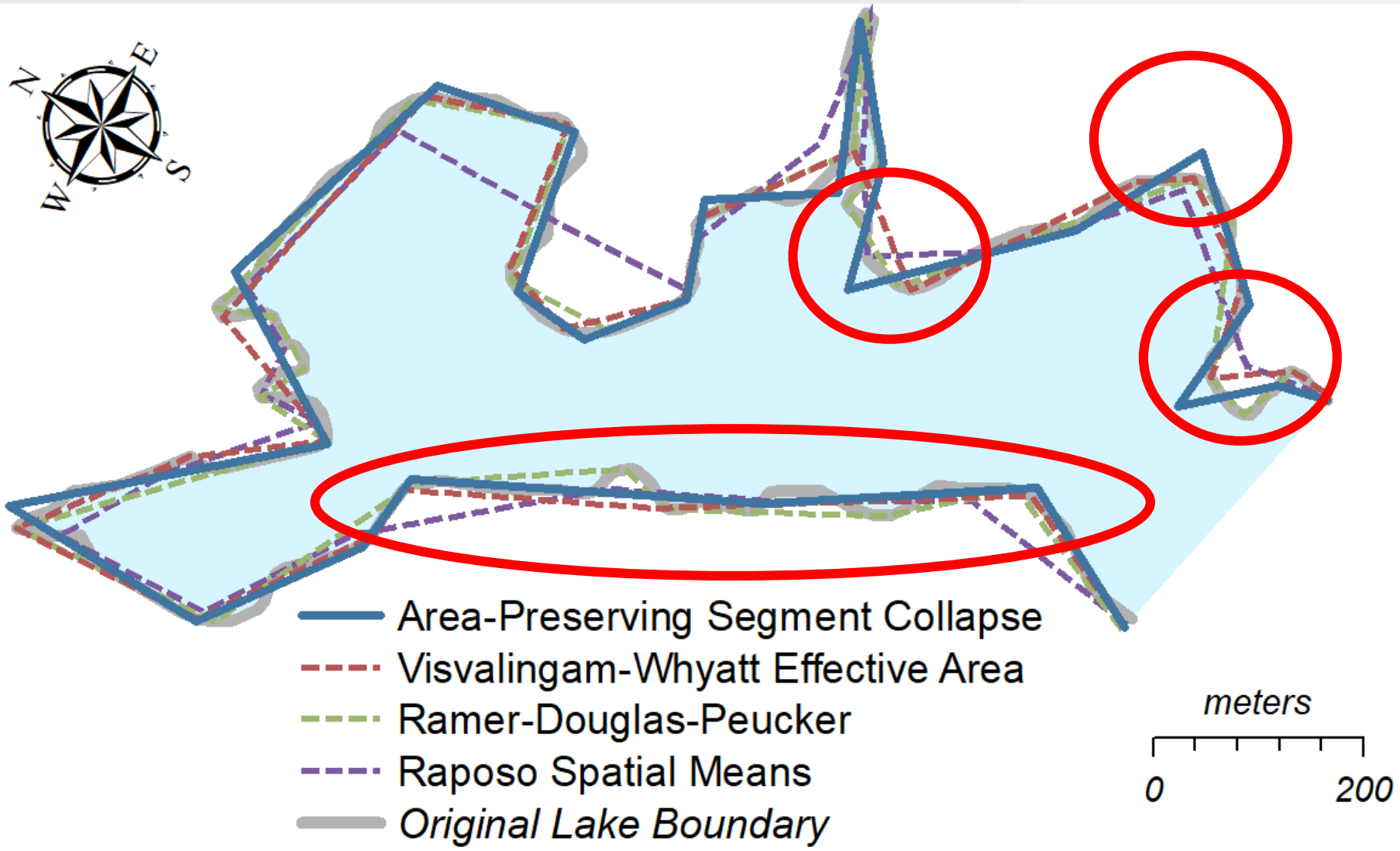
Algorithm Assessment

- Comparison Metrics:
  - area preservation
  - areal displacement
  - modified Hausdorff distance
  - change in boundary complexity
  - topological errors

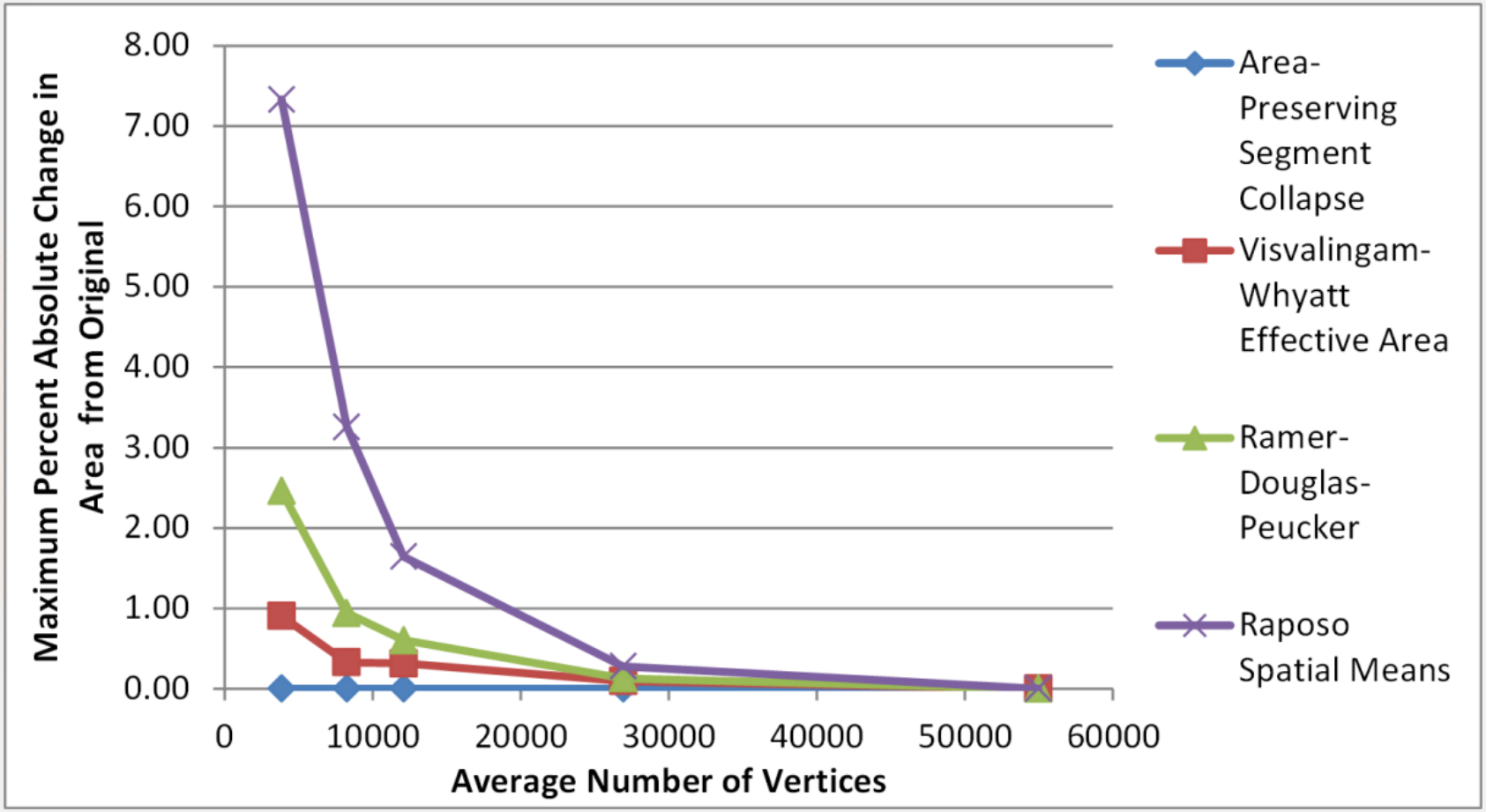


Results: *Visual Comparison*

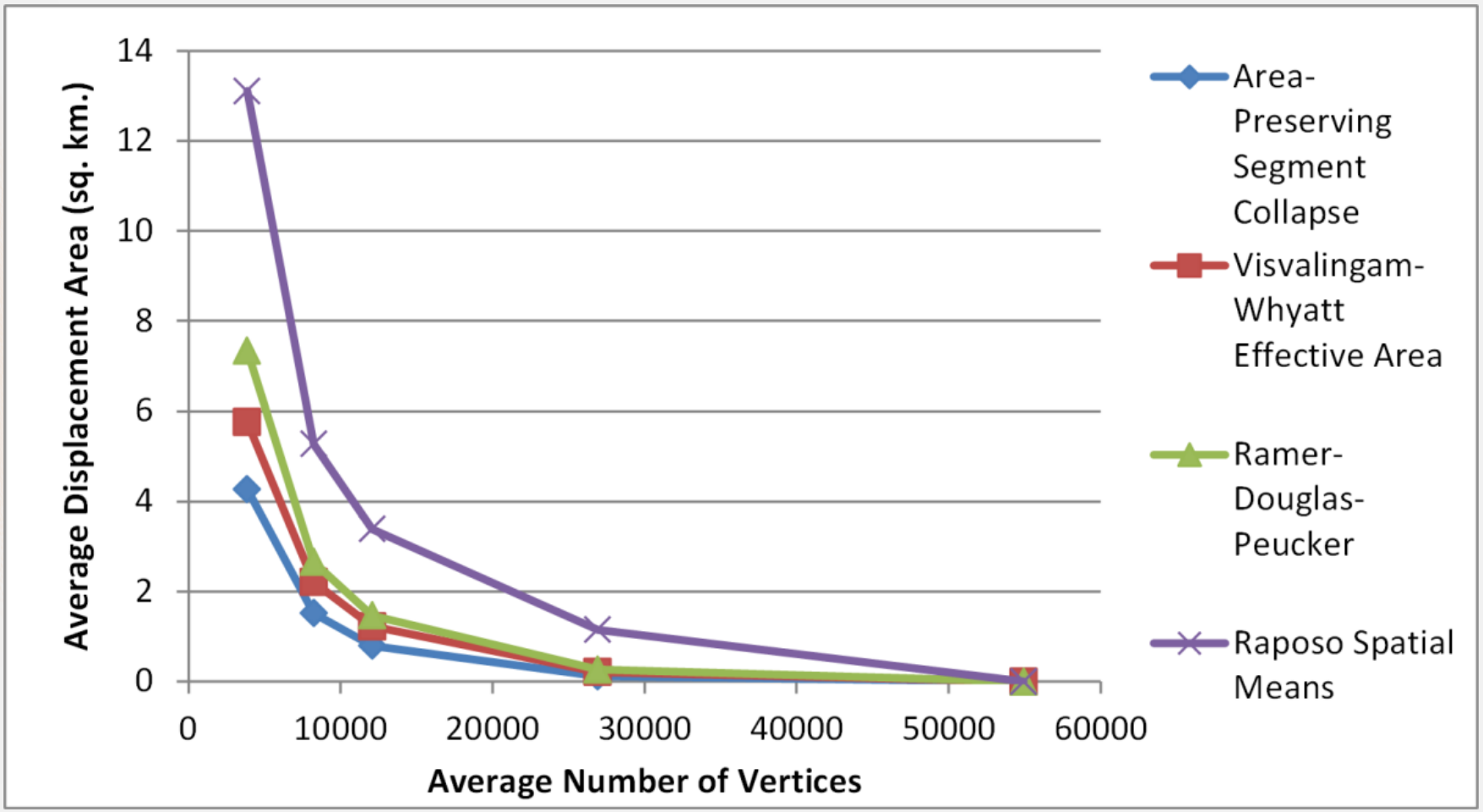
1:1,000,000
(15% of original vertices)



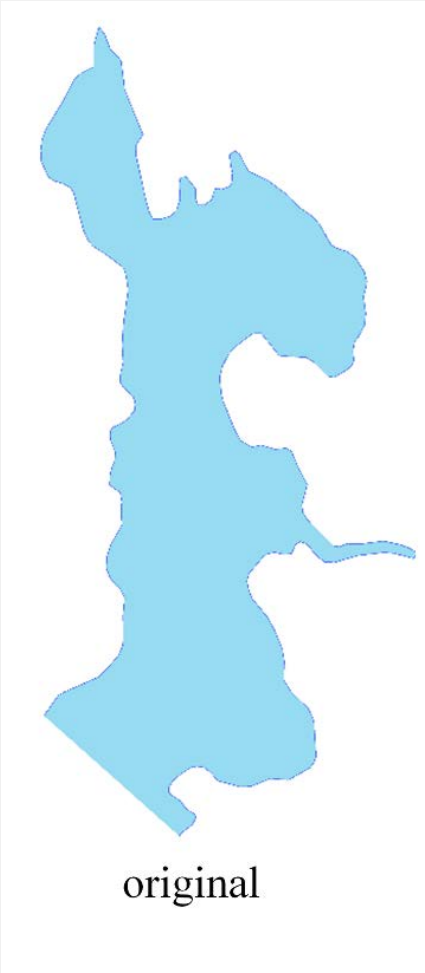
Results: *Area Preservation*



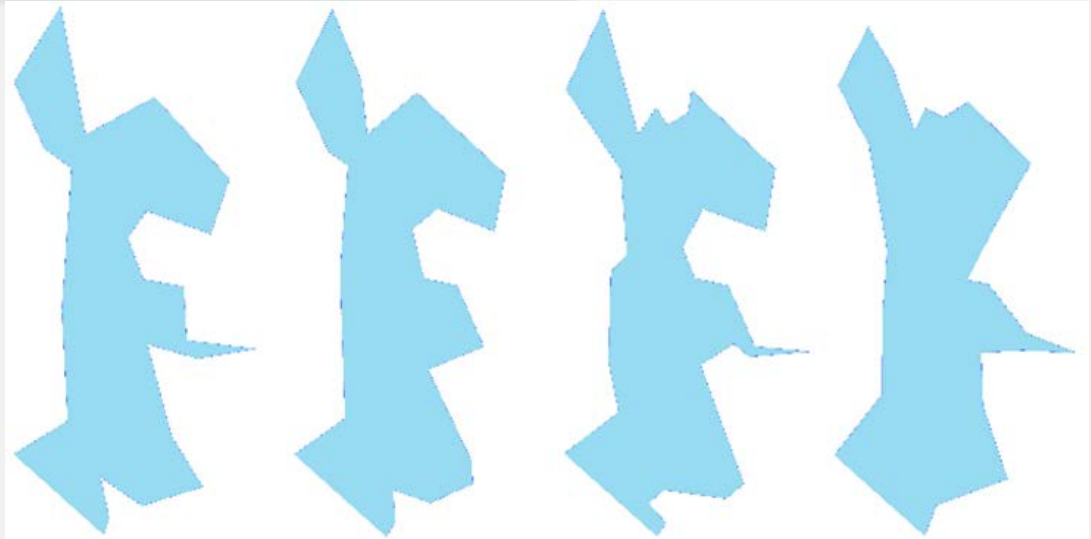
Results: *Areal Displacement*



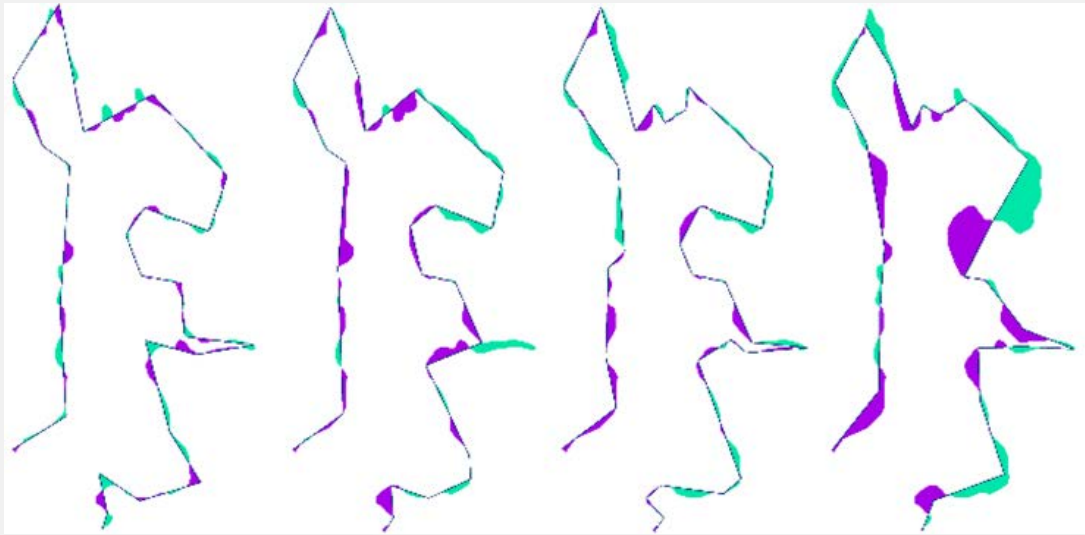
Results: *Areal Displacement*



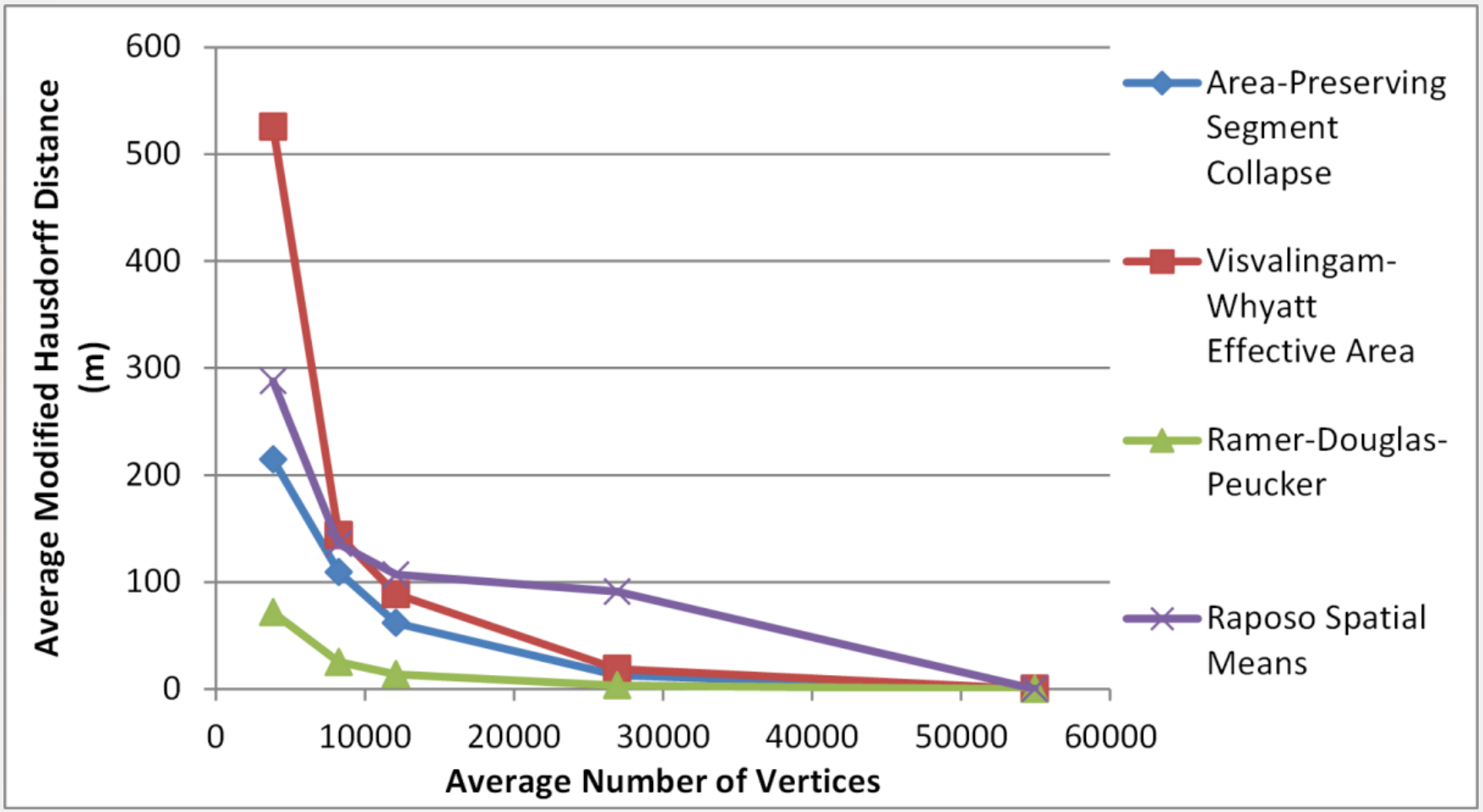
Portion of Lake Winnipesaukee simplified to 1:1,000,000 scale (85% of vertices removed)



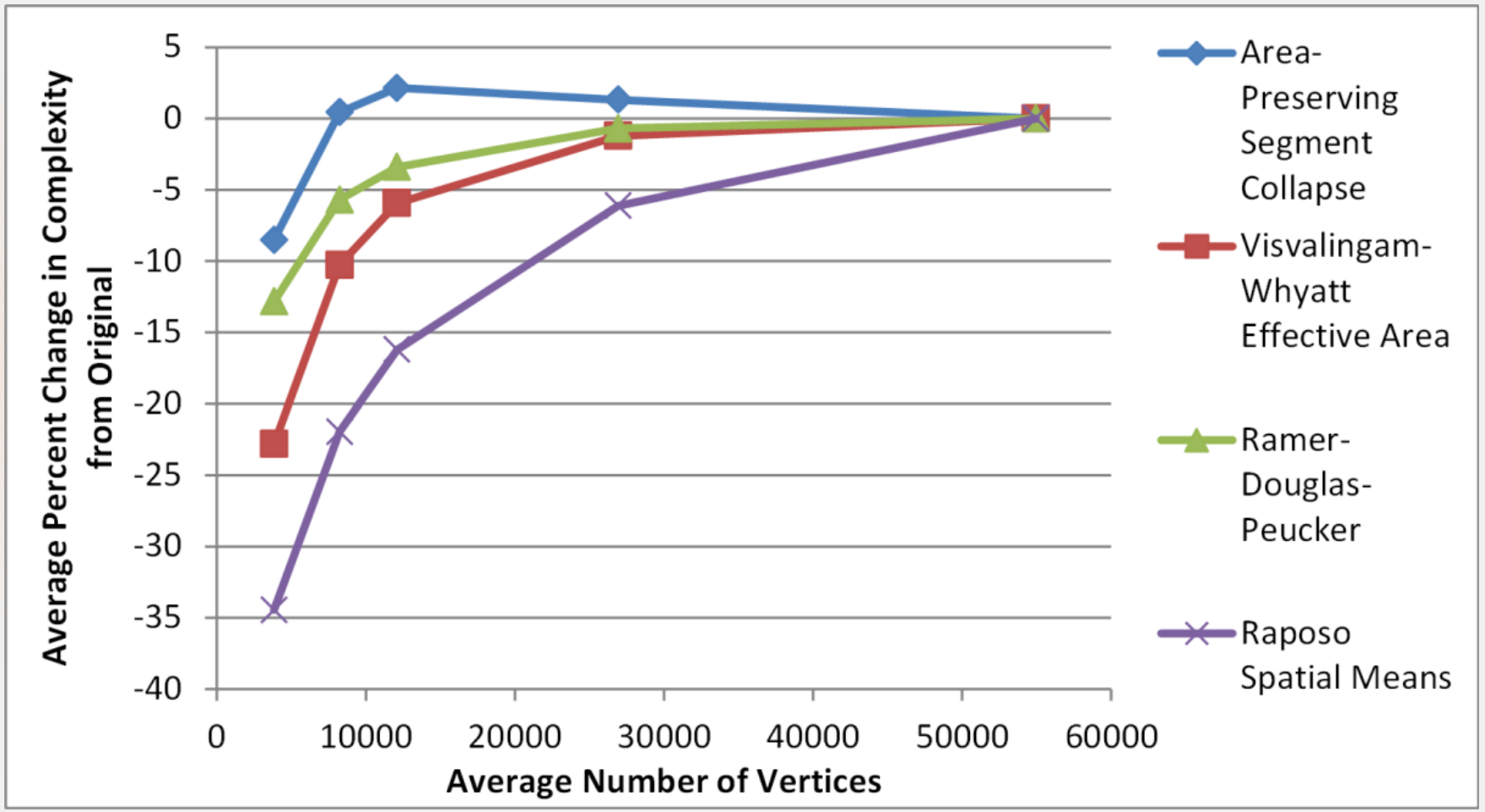
Area-Preserving Segment Collapse | Visvalingam-Whyatt Effective Area | Ramer-Douglas-Peucker | Raposo Spatial Means



Results: *Modified Hausdorff Distance*



Results: *Boundary Complexity*



Results: *Topological Errors*

Simplification level	0.49	0.22	0.15	0.07
Area-Preserving Segment Collapse	4(2)	**67(8)**	**151(9)**	**264(9)**
Visvalingam-Whyatt Effective Area	4(2)	6(3)	21(6)	22(6)
Ramer-Douglas-Peucker	2(1)	14(6)	33(8)	96(6)
Raposo Spatial Means	**7(3)**	28(5)	30(5)	62(7)

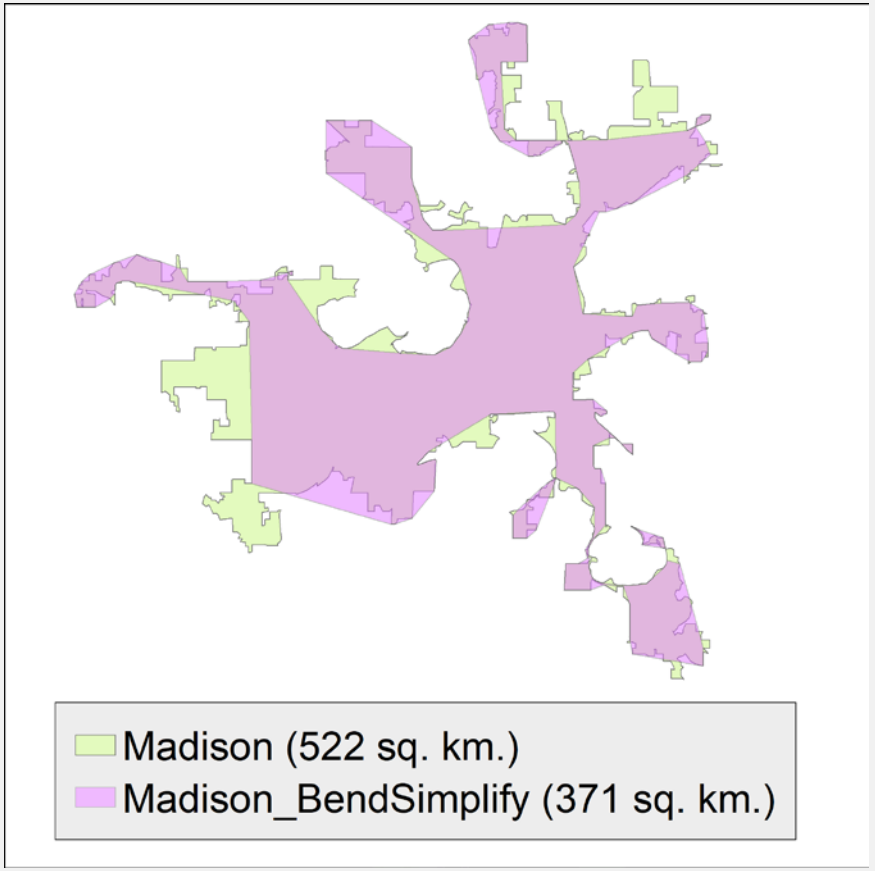
number of self-intersections (number of features with self-intersections)

Summary

- Area preserved exactly ✓
- Areal displacement minimized ✓
- Low modified Hausdorff distance ✓
- Boundary complexity maintained ✓
- High frequency of topological errors ✗

Discussion

- Model-based approach needed to support broad-area modelling
- Area can be preserved exactly with Steiner Points
- Steiner points improve fidelity of simplified line




Questions?